**BHARATIYA VIDYA BHAVAN'S V M PUBLIC SCHOOL, VADODARA**

**QUESTION BANK**

CHAPTER -6 APPLICATIONS OF DERIVATIVES

1. The surface area of a spherical bubble is increasing at the rate of $2cm^2$/sec. Find the rate at which the volume of the bubble is increasing at the instant if its radius is 6 cm.
2. Find the equation of the normal to the curve $x^{2/3} + y^{2/3} = 2$ at (1,1).
3. Find the point on the curve $y^2 = 3x$ for which the abscissa and ordinate change at the same ratio.
4. If y = a logx + b$x^2$+ x has its extreme values at x = -1 and x = 2, then find a and b.
5. Find the intervals in which the function f(x) = $\sin^4 x + \cos^4 x$ , x $\in [0, \pi/2]$ is increasing or decreasing.
6. Find the intervals in which the function f(x) = sin3x, $0 \leq x \leq \frac{\pi}{2}$ is increasing.
7. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.
8. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.
9. If the lengths of three sides of a trapezium, other than the base are equal to 10 cm each, then find the area of trapezium when it is maximum.
10. A window is in the form of a rectangle surmounted by semicircular opening. The total perimeter of the window is 10 m .Find the dimensions of the window to admit maximum light through the whole opening.
11. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.
12. The sum of the surface area of a sphere and cube is given. Show that when sum of their volume is least ,the diameter of sphere is equal to edge of the cube.
13. an open box with a square base is to be made out of a given quantity of cardboard of area $c^2$ square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.
14. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water .Show that the total surface area is least when depth of the tank is half of its width.
15. Find the angle between the curves y = $e^{-x}$ and y = $e^{x}$ at their point of intersection.

16. Find the equation of the normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.
17. Find the slope of the normal at the point $(am^2, am^3)$ for the curve $ay^2 = x^3$.
18. Find the local maxima and minima if any, of the function f(x) = sinx + cosx, $0 < x < \frac{\pi}{2}$. Also, find the local maximum and local minimum values.
19. Find the intervals in which the function f(x) = $10 - 2x^2 - 6x$ is (a) strictly increasing (b) strictly decreasing.
20. Find the intervals in which the function f(x) = $2x^3 + 9x^2 + 12x + 20$ is (a) increasing (b) decreasing.
21. Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line $y - 4x + 5 = 0$
22. An open topped box is to be constructed by removing equal squares from each corner of 3m by 8m rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.
23. Show that a right circular cylinder of a given volume which is open at the top, has minimum total surface area, When its height is equal to the radius of its base.
24. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.
25. Find the local maximum and local minimum, if any, for the function f(x) = $x^3 - 6x^2 + 9x + 15$. Also, find the local extreme values.
26. Find the equation of tangent and normal to the curve x = sin3t , y = cos2t at $t = \frac{\pi}{4}$.
27. For the curve $y = 7x - x^3$, if x increases at the rate of 4 units/second, then how fast is the slope of the curve changing when x=2?
28. An open box with a square base is to be made out of a given quantity of cardboard of area $\boldsymbol{c^2}$ square units. Show that the maximum volume of the box is $\frac{\boldsymbol{c^3}}{\boldsymbol{6\sqrt{3}}}$ cubic units.
29. Find the intervals in which the function f(x) = $(x+1)^3(x-3)^3$ is (a) strictly increasing (b) strictly decreasing.
30. Find the local maxima or minima if any, of the function f(x) = $\sin^4 x + \cos^4 x$, $0 < x < \frac{\pi}{2}$.
31. Find the intervals in which the function f(x) = $x^4 - 8x^3 + 22x^2 - 24x + 21$ is (i) increasing (ii) decreasing
32. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.
33. Water is running out of a conical funnel at the rate of 5 $cm^3$/ sec. If the radius of the cone is 10 cm and altitude is 20 cm, find the rate at which the level of water dropping when it is 5 cm from the top. Explain the importance of water .What measures will you suggest to prevent wastage of water?
34. Show that a cone of greatest volume which can be inscribed in a given sphere is such that three times its altitude is twice the diameter of the sphere. Find the volume of the largest cone inscribed in the sphere of radius R.
35. Show that the tangent lines to the curve $y^2 = 4ax$ at the point x = a are orthogonal.

36. Find the equation of the tangent to the curve $x^2 + 3y = 3$ which is parallel to $y - 4x + 5 = 0$.

37. Prove that the radius of a right circular cylinder of greatest curved surface area, which can be inscribed in a given right circular cone, is half that of the cone.

**38.** Show that the semi vertical angle of the cone of the maximum volume and the given slant height is $\cos^{-1}\left(\frac{1}{3}\right)$.

39. Show that the height of the right circular cylinder of maximum volume that can be inscribed in a cone of height H is $\frac{H}{3}$.

40. Find the intervals which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is (i) Strictly increasing (ii) Strictly decreasing

41. Find the equations of the tangent and normal to the curve $X = a\sin^3\theta$ and $y = a\cos^3\theta$ at $\theta = \frac{\pi}{4}$.

42. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

43. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with vertex at one end of the major axis.

44. Find a point on parabola $y = (x - 3)^2$, where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).

45. The volume of a spherical balloon is increasing at the rate of $25 cm^3/s$. At what rate is the surface area of the balloon increasing when the radius of the balloon is 5cm.

46. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, find the approximate error in calculating its surface area.

47. Find the area of the triangle formed by the tangent and the normal at the point (a,a) on the curve $y^2(2a - x) = x^3$ and the line x = 2a.

48. Find the interval in which the function $f(x) = \frac{4x^2+1}{x}$ is (a) increasing (b) increasing

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