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QUESTION BANK

CHAPTER -4. DETERMINANTS

Answer the following.

1. Find k,If the area of the triangle is 35 square units with vertices (2,-6), (5,4) and (k,4).

2. Write A^{-1} for A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

3. Using the properties of determinants and without expanding , evaluate $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix}$

4. Using properties of determinants, prove the following:

(I) $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$ =(x-y) (y-z) (z-x) (xy + yz + zx).

(II) $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$ = 4 a^2 b^2 c^2

(III) $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$ = (1 + a^2 + b^2 + c^2).

(IV) $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$ = 2 $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

(V) $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

(VI) If a+b+c $\neq$ 0 and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$,then prove a= b = c.

(VII) $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$ = 2(a+b+c) ^{3}

(VIII) $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$

(IX) $\begin{vmatrix} 3a & -a+b & -a+c \\ a-b & 3b & c-b \\ a-c & b-c & 3c \end{vmatrix}$ = 3 (a+b+c)(ab+bc+ca)

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(X) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ = ab + bc + ca + abc

(XI) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ = (a-b)(b-c)(c-a)(a+b+c)

(XII) $\begin{vmatrix} x & x^2 & 1+ax^3 \\ y & y^2 & 1+ay^3 \\ z & z^2 & 1+az^3 \end{vmatrix}$ =(x-y)(y-z)(z-x)(1+a xyz).

(XIII) If none of a, b and c is zero, using properties of determinants, prove that:

$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$ = (bc + ca + ab)^3.

(XIV) In a ΔABC, if $\begin{vmatrix} 1 & 1 & 1 \\ 1+sinA & 1+sinB & 1+sinC \\ sinA+sin^2A & sinB+sin^2B & sinC+sin^2C \end{vmatrix} = 0$,
then prove that ΔABCis an isosceles triangle.

(XV) $\begin{vmatrix} 2y & y-x-z & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} = (x+y+z)^3$.

5. Using matrix method, solve the following system of linear equations:

(i) X−Y+2Z−7 = 0 , 3 X+ 4Y− 5Z+5 = 0, 2X−Y +3Z− 12 = 0 Ans: (x,y,z)=(2,1,3)

(ii) 4X + 3Y + 2Z = 60 , X + 2Y + 3Z = 45 , 6X + 2Y + 3Z = 70 Ans: (x,y,z)=(5,8,8)

(iii) $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$ $\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$ $\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$. x, y, z $\neq 0$.

6. Two Trusts A and B receive Rs. 70,000 and Rs. 55,000 respectively from Central Government to award prize to persons of district in three fields agriculture, education and social service. Trust A awarded 10,

5 and 15 persons in the field of agriculture, education and social service respectively while trust B awarded 15, 10 and 5 persons respectively. If all three prizes together amount to Rs. 6000, then find the amount of each prize by matrix method. What field do you prefer most for award for development of society? Give one reason with justification.

7. The management committee of a residential colony decided to award some of its members for honesty, some for helping others and some others for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice number of awardees for helping others. Using matrix method , find the number of awardees of each category. Apart from these 3 values , suggest one more value which the management committee of the colony must include for awards.

8. Two factories decided to award their employees for 3 values of resourcefulness, competence and determination in the form of Rs x, Rs y and Rs z respectively. The first factory decided to award 4,3 and 2 employees with a prize money of Rs 37,000 and the second factory decided to award 5,3 and 4 employees with a total prize money of Rs 47000. If all the 3 prizes per person together amount to Rs 12000, then using matrix method. Find x , y and z.

9. Two factories decided to award their employees for three values of (a) adaptable to new techniques, (b) careful and alert in difficult situations and (c) keeping calm in tense situations at the rate of Rs x ,Rs y and Rs z per person respectively. The first factory decided to honour respectively 2 , 4 , 3 employees with a total prize money of Rs 29000. The second factory decided the honour respectively 5, 2 and 3 employees with the prize money of Rs 30,500. If the three prizes per person together cost Rs 9500 then represent the above situation by matrix equations and form linear equation using matrix multiplication. Solve these equations by matrix method. Which moral values are reflected in this equation?

10. If A is a square matrix of order 3 such that I adj A I = 64. Find IA'I.

11. Find the value of x from the following: $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0.$

12. Given A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations y + 2z = 7 ; x – y =3 and 2x + 3y + 4z = 17.

13. If A= $\begin{bmatrix} 0 & i \\ i & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then find the value of |2A| + |3B|

14. If A= $\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations:

2x + y + 3z = 3 , 4x – y = 3 , -7x +2y + z = 2

15. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB. Use this to solve the following system of equations : $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$.

16. Find the equation of the line joining A (1 , 3) and B (0 , 0) using determinants and find k if D (k , 0) is a point such that area of Δ ABD is 3 sq. units.

17. Using matrix method, solve : $x - y + 2z = 1$, $2y - 3z = 1$, $3x - 2y + 4z = 2$.

18. Solve for x, when $\begin{vmatrix} 15 - 2x & 11 & 10 \\ 11 - 3x & 17 & 16 \\ 7 - x & 14 & 13 \end{vmatrix} = 0$.

19. Using determinant, find the area of the triangle whose vertices are (3,8),(-4,2) and (5,-1).

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