

QUESTION BANK

CHAPTERS – 3 MATRICES

Answer the following.

1. Construct a 3 X 4 Matrix $A = [a_{ij}]$, whose elements a_{ij} are given by: $a_{ij} = \frac{2i-3j}{2}$
2. Construct 3x4 matrix, whose elements are given by $a_{ij} = 2i-3j$
3. Construct a 3 x 4 matrix , whose elements a_{ij} are given by $a_{ij} = \frac{1}{2} |-3i + j|$.
4. Find X and Y, if $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$
5. If matrix $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, then find AA^T . Ans. $\begin{bmatrix} 14 \end{bmatrix}$
6. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, find value of k. Ans. $k = 17$
7. For what value of x, matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ is the skew symmetric matrix? Ans. $X = 2$
8. Find the value of x, if $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$. Ans. $X = 2$ or $X = -14$
9. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find matrix D such that $CD - AB = O$.

Ans. $\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$

10. Express matrix $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrix and

verify your result.

11. If $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, find $f(A)$, where $f(x) = x^2 - 5x + 7$. Ans. $\begin{bmatrix} -15 & -20 \\ 20 & 15 \end{bmatrix}$

12. If $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, then for what value of α is A an identity matrix?

13. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$.

14. A is a square matrix of order 3 and $|A| = 7$. Write the value of $|\text{adj. } A|$.

15. If $A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2,

show that $(I+A)(I-A) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

16. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, Verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1} .

17. Express $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix and verify your result.

18. Prove the following by using the principle of mathematical induction. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then $A^n = \begin{bmatrix} 1 + 2n & -4n \\ n & 1 - 2n \end{bmatrix}$ for every positive integer n.

19. If $A = \begin{bmatrix} 0 & i \\ i & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then find the value of $|2A| + |3B|$.

20. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, show that $A^3 - 6A^2 + 5A + 11I = O$. Hence find A^{-1} .

21. Express $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

22. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, find a and b. (a=1, b=4)

23. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ and $f(x) = x^2 - 5x - 14$, find $f(A)$.

24. Find x, y, a and b if: $\begin{bmatrix} 2x - 3y & a - b & 3 \\ 1 & x + 4y & 3a + 4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$

25. If $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, Find matrix C such that $5A + 3B + 2C$ is a null matrix.

26. If $A = \begin{bmatrix} \cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$ find the value of A^2 .

27. For what values of x and y are the following matrices A and B equal?

$$A = \begin{bmatrix} 2x + 1 & 2y \\ 0 & y - 5y \end{bmatrix}, B = \begin{bmatrix} x + 3 & y + 2 \\ 0 & -6 \end{bmatrix}.$$

28. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, find x, y such that $(xI + yA)^2 = A$.

29. If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then evaluate $A^2 - 7A + 10I$.

30. If $\begin{bmatrix} x - y & z \\ 2x - y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x^2 + y^2$.

31. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x .

32. Using elementary transformation, find the inverse of the following matrices.

(i) $A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$ Ans. $\begin{bmatrix} -2 & 1/2 & 1 \\ 11 & -1 & -6 \\ 4 & -1/2 & -2 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 1 \\ 1 & 6 & 3 \end{bmatrix}$ Ans. $\begin{bmatrix} 2/21 & 8/21 & -1/3 \\ -5/21 & 1/21 & 1/3 \\ 2/3 & -1/3 & -1/3 \end{bmatrix}$

33. Express $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

34. Using elementary transformation, find the inverse of $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$.

35. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, find x, y such that $(xI + yA)^2 = A$.

36. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Verify that $A^3 - 6A^2 + 9A - 4I = O$. hence find A^{-1} .

37. Find the values of x, y, z , if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies the equation $A'A = I$.
38. Given $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$. Using mathematical induction, prove that $A^n = \begin{bmatrix} 1 + 2n & -4n \\ n & 1 - 2n \end{bmatrix}$.
39. Find a matrix X , such that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.
40. For what value of x , is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew – symmetric matrix?
41. Obtain the inverse of the following matrix, using elementary operations:
42. $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$
43. Simplify : $\tan \theta \begin{bmatrix} \sec \theta & \tan \theta \\ \tan \theta & -\sec \theta \end{bmatrix} + \sec \theta \begin{bmatrix} -\tan \theta & -\sec \theta \\ -\sec \theta & \tan \theta \end{bmatrix}$
44. If the points (a_1, b_1) , (a_2, b_2) and $(a_1 + a_2, b_1 + b_2)$ are collinear then show that $a_1 b_2 = a_2 b_1$.
45. If B is a skew symmetric matrix, write whether (ABA') is symmetric or skew symmetric
46. If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, find x , $0 < x < \frac{\pi}{2}$ when $A + A' = I$.
47. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$, $n \in \mathbb{N}$
48. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$, $n \in \mathbb{N}$.
49. 4. If $\begin{bmatrix} x - y & z \\ 2x - y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x^2 + y^2$.
50. 5. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x .
