

QUESTION BANK

CHAPTER – 2 INVERSE TRIGONOMETRIC FUNCTIONS

Answer the following.

- Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, $-\frac{\pi}{2} < x < \frac{3\pi}{2}$ in the simplest form.
- Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \sin^{-1}\left(\sin\frac{7\pi}{6}\right)$.
- Solve for x: $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$.
- Prove: $\tan^{-1}\left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$
- $\tan\left[\frac{\pi}{4} + \frac{1}{2} \cos^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2} \cos^{-1}\left(\frac{a}{b}\right)\right] = \frac{2b}{a}$.
- Simplify: $\tan\left[\frac{1}{2} \cos^{-1}\frac{\sqrt{5}}{3}\right]$.
- Find the principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.
- Simplify: $\tan^{-1}\left(\frac{3 \sin 2x}{5 + 3 \cos 2x}\right) + \tan^{-1}\left(\frac{\tan x}{4}\right)$.
- If $\cos^{-1}\left(\frac{x}{a}\right) + \cos^{-1}\left(\frac{y}{b}\right) = \alpha$, Prove that $\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} = \sin^2 \alpha$
- Solve for x : $\sin^{-1}(1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$
- Prove : $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2} \cos^{-1}\left(\frac{3}{5}\right)$
- Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.
- Prove that $\sin^{-1} \frac{3}{5} - \sin^{-1} \frac{8}{17} = \cos^{-1} \frac{84}{85}$.
- If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, where $xy < 1$, find the value of $x+y+xy$.
- Find x, if $\tan^{-1} 3 + \cot^{-1} x = \frac{\pi}{2}$.
- Prove: $\cos\left(\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{2}\right) = \frac{6}{5\sqrt{13}}$
- Solve for x: $2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x)$, $0 < x < \frac{\pi}{2}$.
- Solve for x: $\cos^{-1} x + \sin^{-1} \frac{x}{2} = \frac{\pi}{6}$.
- Show that : $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = 2(\tan^{-1} 1 + \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3})$
- If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$.
- Evaluate : $\sin^{-1}(\cos(-105^\circ))$.
- Solve for x : $\sin^{-1} \frac{2a}{1+a^2} + \sin^{-1} \frac{2b}{1+b^2} = 2 \tan^{-1} x$, $ab < 1$
- Prove: $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$
- Write the principal value of $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$.
- Solve for x: $2 \tan^{-1}(\sin x) = \tan^{-1}\left(\frac{2}{\cos x}\right)$, $x \neq \frac{\pi}{2}$.
- Find the value of $\sin^{-1}\{\cos(\sin^{-1} x)\} + \cos^{-1}\{\sin(\cos^{-1} x)\}$.

27. Prove: $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$
28. Prove: $\tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{4}$
29. Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$
30. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, prove that $xy + yz + zx = 1$.
31. Solve for x: $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$
32. Prove that : $2 \tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2 \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$
33. Prove that: $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1}\frac{1}{3} = \frac{9}{4} \sin^{-1}\frac{2\sqrt{2}}{3}$
34. Write in simplest form : $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \pi$.
35. Prove : $2 \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$
36. Prove that $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$
37. Solve for x : $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$.
38. Prove that $\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \pi/4$.
39. If $\cos^{-1}(x/a) + \cos^{-1}(y/b) = \theta$, prove that $\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \theta + \frac{y^2}{b^2} = \sin^2 \theta$.
40. Prove : $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$
41. Prove that : $4(\cot^{-1} 3 + \operatorname{cosec}^{-1} \sqrt{5}) = \pi$
42. Solve for x : $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$
43. Write the principal value of $\tan^{-1}[\sin(-\pi/2)]$.
