

QUESTION BANK

CHAPTER -10 . VECTORS

ANSWER THE FOLLOWING QUESTIONS:

1. Find $|\vec{x}|$, if for a unit vector \vec{a} , $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 15$.
2. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector \vec{p} which is perpendicular to both \vec{a} and \vec{b} and $\vec{p} \cdot \vec{c} = 1$
3. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$.
4. For what value of μ are the vectors $\vec{a} = 2\hat{i} + \mu\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ perpendicular to each other?
5. Find the angle between two vectors \vec{a} and \vec{b} if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} \times \vec{b}| = 6$.
6. Find the direction cosines of a line, passing origin and lying in the first octant, making equal angles with the three coordinate axes.
7. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ .
8. If \vec{a} is a unit vector perpendicular to \vec{b} and $(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b}) = -10$, find $|\vec{b}|$.
9. Find the projection of \vec{AB} on \vec{CD} , where A(4 - 3, 2), B(1, -1, -1), C(2, 2, 2) and D(3, 3, 3).
10. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector \vec{c} such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$
11. If \vec{a} and \vec{b} are the position vectors of the points (1, -1) and (-2, m) respectively. Find the value of m for which \vec{a} and \vec{b} are collinear.
12. If a unit vector \vec{a} makes angle $\frac{\pi}{4}$ with \hat{i} , $\frac{\pi}{3}$ with \hat{j} and an acute angle θ with \hat{k} , then find the component of \vec{a} and the angle θ .
13. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.
14. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector \vec{p} which is perpendicular to both \vec{a} and \vec{b} and $\vec{p} \cdot \vec{c} = 18$.
15. If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = 2\hat{j} - \hat{k}$ are three vectors, find the area of the parallelogram having diagonals $(\vec{a} + \vec{b})$ and $(\vec{b} + \vec{c})$.
16. If the three vectors a, b and c are coplanar prove that the vectors a + b, b + c, c + a are also coplanar.
17. Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.
18. Find the value of λ if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.
19. Find the direction cosines of the line passing through the points (-2, 4, -5) & (1, 2, 3).

20. Let $\vec{a} = 2\vec{i} + 2\vec{j} + \vec{k}$, $\vec{a} \cdot \vec{b} = 14$, $\vec{a} \times \vec{b} = 3\vec{i} + \vec{j} - 8\vec{k}$. Find \vec{b} .
21. If the vectors $a\vec{i} + a\vec{j} + c\vec{k}$, $\vec{i} + \vec{k}$ and $c\vec{i} + c\vec{j} + b\vec{k}$ are coplanar, show that $c^2 = ab$.
22. If \vec{a} and \vec{b} are two unit vectors and θ is the angle between them, then find the value of θ such that $\frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|}$ is a unit vector.
23. The magnitude of the vector product of the vector $\vec{i} + \vec{j} + \vec{k}$ with the unit vector along the sum of vectors $\vec{i} + 4\vec{j} - 5\vec{k}$ and $\lambda\vec{i} + 2\vec{j} + 3\vec{k}$ is equal to $\sqrt{2}$. Find the value of λ .
24. Show that three vectors $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} - 3\vec{j} - 5\vec{k}$ and $\vec{c} = 3\vec{i} - 4\vec{j} - 4\vec{k}$ are coplanar.
25. Find the projection of vectors \vec{a} on \vec{b} if $\vec{a} \cdot \vec{b} = 8$ and $\vec{b} = 2\vec{i} + 6\vec{j} + 3\vec{k}$.
26. If $\vec{a} = 5\vec{i} - \vec{j} - 3\vec{k}$ and $\vec{b} = \vec{i} + 3\vec{j} - 5\vec{k}$, then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.
27. Find the vector and Cartesian equation of the plane passing through the points A(2,2,-1), B(3,4,2) and C(7,0,6).
28. Evaluate the value of $\vec{i} \cdot (\vec{j} \times \vec{k}) + \vec{j} \cdot (\vec{k} \times \vec{i}) + \vec{k} \cdot (\vec{i} \times \vec{j})$,
29. Prove that $\vec{a} \cdot (\vec{b} \times \vec{c}) \times \left(\frac{\vec{a} + \vec{b} + \vec{c}}{|\vec{a} + \vec{b} + \vec{c}|} \right) = 0$.
30. Find the value of λ so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.
