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QUESTION BANK

CHAPTERS – 1 RELATIONS AND FUNCTIONS

Answer the following.

1. Let f and g be two real functions defined as f(x) = 3x-2 and g(x) = $\frac{2+x}{3}$. Find fog and gof. Can we say that one is inverse of the other?

2. Consider f : $R_+ \rightarrow [-9,\infty)$ given by f(x) = $5x^2 + 6x - 9$. Prove that f is invertible. Also, find f^{-1}.

3. On the set R-{-1} a binary operation*is defined by a * b = a + b + ab for all a,b ∈ R-{-1}. Prove that * is commutative as well as associative on R-{-1}. Find the identity element, if exist. Also, prove that every element of R-{-1 }is invertible.

4. If f(x) = $x^2 + 1$ and g(x) = $\frac{1}{x-1}$, find gof (-5).

5. Let A = WxW and * be the binary operation on A defined by (a,b)*(c,d) = (a+c, b+d). Show that * is commutative and associative. Find the identity element for * on A, if any.

6. Let N be the set of all natural numbers and R be the relation on N x N defined by (a,b)R(c,d) iff ad (b+c) = bc (a+d). Show that R is an equivalence relation.

7. If the binary operation * on the set of integers is defined as a * b = $a+3b^2$, find the value of 8 * 3.

8. If the function f: R→R is given by f(x)= $x^2 + 3x + 1$ and g: R → R is given by g(x) = 2x – 3 ,find fog and gof .

9. Let Z be the set of integers and R be the relation on Z defined as R = {(a,b): a – b is divisible by n, a,b ∈ Z}. Prove R is an equivalence relation.

10. Let * be a binary operation on Q defined by a* b =$\frac{3ab}{5}$. Show that * is commutative as well as associative. Also find the identity element if it exists.

11. Show that the relation S on the set of real numbers, defined as S = {(a,b) : $a \le b^3$, a,b ∈R } is neither reflexive nor symmetric nor transitive .
12. Show that the relation R in the set A = {x: x ∈ Z ,0 ≤ x ≤ 12 } given by R = {(a,b) : |a – b | is divisible by 4 } is an equivalence relation . Find the set of all elements related to 1.

13. Find the number of relations from A to B ,if n(A) = 3 and n (B) = 2 .
14. Let X be a non-empty set. P (X) be its power set. Let ∗ be an operation defined on elements of P (X) by , A ∗ B = A ∩ B $\forall A, B \in P(X)$. (i) Prove that ∗ is a binary operation in P (X).

(ii) Is $*$ commutative? (iii) Is $*$ associative? (iv) Find the identity element in P (X) w.r.t $*$
(v) Find all the invertible elements of P (X).

15. Find the inverse of the function f: N→N defined by $f(x)= x^2 + x + 1$, $x \in N$, if it exists.

16. If f: R → R and g : R → R defined by $f(x)= 2x^2 +3x + 5$ and $g(x) = 4x – 5$,find (g o f)(-3).

17. Let A = R – {2} and B = R – {1}. If f: A → B is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is bijective. Hence find f^{-1}.

18. If A = N X N and * on A is defined by (a , b)*(c,d)= (ad+ bc , bd) for all (a,b),(c,d) ϵ A, then show that * is (i) commutative (ii) associative (iii) find identity element if any.(iv) find inverse.

19. Consider f: $R_+ \rightarrow [-5, \infty)$ given by $f(x)= 9x^2 + 6x – 5$ .Show that f is invertible. Find the inverse of f .

20. If f: R → R be given by $f(x) = \sin^2 x + \sin^2 (x+\frac{\pi}{3}) + \cos x \cos (x+\frac{\pi}{3})$ for all $x \in R$ and
g : R → R be such that g (5/4) = 1, then prove that g o f : R → R is a constant function.

21. Let I be the set of integers and a relation R on I be defined by $a^b = b^a$ for all a,b $\in$ I . Show that R is an equivalence relation

22. If $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x-1}$, find gof (5).
23. Consider f:R→R given by f(x) = 4x+3. Show that f is invertible. Find the inverse of f.

24. Consider the binary operations $*$: R×R and o: R×R defined as $a*b = |a-b|$ and aob = a, for all a,b ∈ R. Show that $*$ is commutative but not associative. o is associative but not commutative.

25. Consider f : $R_+ \rightarrow [4,\infty)$ given by $f(x) = x^2+4$, where R_+ is the set of all non-negative numbers. Show that f is invertible and find its inverse.

26. Let A = N x N and * be the binary operation on A defined by $a * b = \frac{a+b}{2}$ for all a,b ϵ N.
Is * commutative and associative. Find the identity element for * on A, if any.

27. Let N be the set of all natural numbers and R be the relation on N x N defined by (a,b) R (c,d) if and only if, ad (b+c) = bc (a+d). Show that R is an equivalence relation.

28. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, prove that $f\left(\frac{2x}{1+x^2}\right) = 2 f(x)$.

29. Let A = Q x Q and * on A is defined by (a,b) *(c,d) = (ac, b +ad) for all (a,b), (c,d) ϵ A, then

(i) Show that * is a binary operation on A. (ii) Show that operation * is non-commutative on A. (iii)Show that operation * is associative on A. (iv) Find the identity element for operation * on A, if any. (v) Find the invertible elements of operation * on A.

30. Let A = WxW and * be the binary operation on A defined by (a,b)*(c,d) = (a+c, b+d). Show that * is commutative and associative. Find the identity element for * on A, if any.

31. Let N be the set of all natural numbers and R be the relation on N x N defined by (a,b) R (c,d) ⇔ ad = bc for all (a,b), (c,d) ∈ N x N. Prove that R is an equivalence relation.

32. If f: R→R is defined by f(x)=$(3-x^3)^{\frac{1}{3}}$, then find f∘f(x) .

33. If the binary operation * on the set Z of integers is defined by a*b =a+b-5, then write the identity element for the operation * in Z.

34. The binary operation *:R×R→R is defined as a*b =2a+b. Find (2*3)*4.

35. Let f be the greatest integer function and g be the absolute value function, then find g o f (7/5) - f o g (- 7/ 5)

36. Find the number of binary operations on a set A where n (A) = 3 .

37. A relation R on the set of all non-zero complex numbers is defined by z_1R z_2 iff $\left(\frac{z_{1-z_2}}{z_{1+z_2}}\right)$ is real. Show that R is an equivalence relation.

38. Define (i) injective function (ii) many –one function (iii) into function (iv) bijective function .

Also give suitable example for each function.

39. Let a relation R_1 on the set R of real numbers be defined as (a,b)∈R_1 ⇔ 1 + ab>0 for all a, b ∈ R.

Show that R_1 is reflexive and symmetric but not transitive.

40. Examine if * on Z defined by a*b = a^b , for every a , b ∈Z is binary or not. Discuss the case when

Z is replace by N.

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