BHARATIYA VIDYA BHAVAN'S V.M.PUBLIC SCHOOL, VADODARA

STD: XII SAMPLE PAPER: 9 MAX MARKS:100

MATHEMATICS TIME : 3 HRS

. General Instructions:

- All questions are compulsory
- The question paper consists of 29 questions divided into three sections A, B , C and D.
- Section A contains 4 questions of 1 mark each.
- Section B contains 8 questions of 2 marks each.
- Section C contains 11 questions of 4 marks each.
- Section D contains 6 questions of 6 marks each.

SECTION A

1. If $A+B=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ and $A-2B=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$, then find A.
2. If A and B are two non-singular matrices such that $|A|=7$ and $|B|=5$ then find the value of $|B^{-1}.A.B|$.
3. Differentiate sin ^{2} x with respect to e $^{\cos x}$
4. Evaluate: $\frac{d}{dx}\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right)$.

SECTION B

5. If $A=\begin{bmatrix}x & 3x-y\\ 2x+z & 3y-w\end{bmatrix}=\begin{bmatrix}3 & 2\\ 4 & 7\end{bmatrix}$, then find w, x ,y and z.
6. Solve for x : $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$.
7. Verify Rolles theorem for the function f(x) = x ^{2}+ 2 x - 8, x$\in$ [– 4, 2]
8. Evaluate : $\int_{-1}^{1} e^{|x|}.dx$.
9. Find the differential equation representing the family of curves $y=e^{c.x}$
10. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them , then find the value of θ such that $\vec{a}+\vec{b}$ is a unit vector.
11. If $P(A)=0.4$, $P(B)=0.8$ and $P(B/A)=0.6$, then find $P(A/B)$.
12. A problem is given to two students. The probabilities their solving it are $1/2$ and $1/3$. Find the probability of both solving it.

SECTION.C

13. Prove that the curves y^2 = 4ax and x y = c^2 cut at right angles if c ^{4} = 32 a ^{4}

OR

Find the equation of the tangent to the curve y = x^2 - 2x +7 which is perpendicular to the line 5y – 15x =13.

14. Prove using properties that $\begin{vmatrix}(b+c)^2 & a^2 & a^2\\ b^2 & (c+a)^2 & b^2\\ c^2 & c^2 & (a+b)^2\end{vmatrix}=2abc(a+b+c)^3$.
15. If $y=x^x$, prove that $\frac{d^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=0$.
16. Prove that: $\int_a^b f(x).dx = \int_a^b f(a+b-x).dx$. Hence evaluate : $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$.

OR

Evaluate: $\int_0^2 |x^2 + 2x - 3|.dx$

17. Let X be a non-empty set. P (X) be its power set. Let $*$ be an operation defined on elements of P (X) by , A $*$ B = A $\cap$ B $\forall A, B \in P(X)$. Then,

i) Prove that $*$ is a binary operation in P (X).
ii) Is $*$ commutative?
iii) Is $*$ associative?
iv) Find the identity element in P (X) w.r.t $*$
v) Find all the invertible elements of P (X)

18. If the vectors a i + a j +c k, i +k and c i + c j+ b k are coplanar ,show that $c^2 = ab$.

19. Evaluate:. $\int \frac{x+3}{x^2 - 2x - 5} dx$

20. Solve the differential equation: $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$.

OR

Solve the differential equation: $(3xy + y^2)dx + (x^2 + xy)dy = 0$.

21. Find the Cartesian equation of the plane passing through the points A(0, 0, 0) and B(3, –1, 2) and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$

22. A man is known to speak truth 4 out of 5 times. He throws a pair of dice and reports that it is a doublet. Find the probability that it is actually a doublet.

23. Evaluate: $\int \frac{dx}{\cos x.(5 - 4\sin x)}$.

SECTION D

24. If the length of three sides of a trapezium other than base are equal to 10 cm, then find the area of the trapezium when it is maximum.

OR

Show that the rectangle of maximum area that can be inscribed in a circle is a square.

25. Find the area bounded by the lines x + 2y = 2 ; y – x =1 and 2x + y = 7

OR

Find the area of that part of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

26. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contain 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

27. Given that A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ find AB. Hence using this product to solve the system of equations : x – y + z = 4, x – 2y – 2z = 9, 2x + y + 3z = 1.

28. In a bolt factory, machines A, B and C, manufacture respectively 25%, 35%, 40% of the total bolts. Of their output 5%, 4% and 2% respectively are defective bolts. A bolt is drawn at random and is found to be defective. Find the probability that it is manufactured by machine B

29. Find length and foot of the perpendicular from the point P(1, 2, 3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, find the image of the point in the line.

OR

Find the equation of the plane passing through the intersection of the planes

2x + y – z = 3 and 5x– 3 y + 4z + 9 = 0 and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{4}$.

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