BHARATIYA VIDYA BHAVAN'S V.M.PUBLIC SCHOOL, VADODARA

STD: XII **SAMPLE PAPER: 8** **MAX MARKS:100**
MATHEMATICS **TIME : 3 HRS**

General Instructions:

- All questions are compulsory
- The question paper consists of 29 questions divided into three sections A, B , C and D.
- Section A contains 4 questions of 1 mark each.
- Section B contains 8 questions of 2 marks each.
- Section C contains 11 questions of 4 marks each.
- Section D contains 6 questions of 6 marks each

SECTION – A

1. If $f : R \to R$ be defined by $f(x) = (3 - x^3)^{1/3}$, then find *fof* (*x*).

2. Find the area of the triangle whose vertices are P(1, 1), Q(2, 7) and R(10, 8).

3. Find the principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.

4. Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

SECTION – B

5. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, show that $A^{-1} = \frac{1}{19} A$.

6. Evaluate: $\int \sin 3x \cos 4x \, dx$

7. Find the value of λ if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

8. Evaluate : $\int_0^{\pi} \frac{x}{1 + \sin x} dx$

9. Find the direction cosines of the line passing through the points (–2, 4, –5) & (1, 2, 3).

10. Discuss the commutativity & associativity of binary operation * defined on Q by the rule a * b = a – b + a b for all a, b ε Q.

OR

Show that the relation R in the set $A = \{x : x \in W, 0 \le x \le 12\}$ given by $R = \{ (a, b) :$ (*a*, *b*) is a multiple of 4} is an equivalence relation. Find the set of all elements related to 2.

11. Two balls are drawn from an urn containing 2 white, 3 red and 4 black balls one by one without replacement. What is the probability that at least one ball is red?

12. Prove that : $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = (bc+ca+ab+abc)$

SECTION C

13. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$

14. Solve of *x* : $\tan^{-1}\left(\dfrac{x-1}{x-2}\right) + \tan^{-1}\left(\dfrac{x+1}{x+2}\right) = \dfrac{\pi}{4}, \quad |x| < 1$

OR

Solve for *x* : $\cos^{-1}\left(\dfrac{x^2-1}{x^2+1}\right) + \tan^{-1}\left(\dfrac{2x}{x^2-1}\right) = \dfrac{2\pi}{3}$.

15. For what value of *k* is the following function continuous at *x* = 0?

$$f(x) = \begin{cases} \dfrac{1-\cos 4x}{8x^2}, & if \ x \neq 0; \\ k, & if \ x = 0 \end{cases}$$

16. Evaluate : $\displaystyle\int \frac{1}{\sqrt{8+2x-x^2}} \, dx$

17. Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$.

18. Differentiate $x^{\sin x} + (\sin x)^{\cos^{-1} x}$ with respect to *x*.

OR

If $y = \tan^{-1}\left\{\dfrac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right\}$, find $\dfrac{dy}{dx}$.

19. Find a point on parabola $y = (x-3)^2$, where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).

OR

The volume of a spherical balloon is increasing at the rate of 25cm^3/s. At what rate is the surface area of the balloon increasing when the radius of the balloon is 5cm.

20. Solve : $\dfrac{dy}{dx} + \dfrac{y}{x} = e^x$

21. Solve : $(x^2 + xy) \, dy = (x^2 + y^2) \, dx$

22. Find the shortest distance between the lines, whose equations are $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{10-z}{-7}$ and $\frac{x-15}{3}=\frac{58-2y}{-16}=\frac{z-5}{-5}$.

SECTION – D

23. Using matrices solve the following system of equations:

$x+2y-3z=-4;\quad 2x+3y-3z=2;\quad 3x-3y-4z=11$

24. Find the equation of the plane passing through the point A (1, 1, –1) perpendicular to the planes $x+2y+3z-7=0$ and $2x-3y+4z=0$.

25. Using integration, find the area of the region enclosed between the curve $y^2 = x$ and the line $x + y = 2$.

26. Evaluate : $\int_1^4 \left[|x-1|+|x-2|+|x-4|\right]\,dx$

OR

Evaluate : $\int \frac{e^x\,dx}{e^{2x}+6e^x+5}$

27. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

OR

Show that the height of a closed cylinder of given volume and the least surface area is equal to its diameter.

28. A manufacturer produces two types of steel trunks. He has two machines A and B. the first type of trunk requires 5 hours on machine A and 3 hours on machine B. The second type of trunk requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs 30 and Rs 25 per trunk on the first type and second type respectively. How many trunks of each type must be make each day to make the maximum profit.

29. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus and scooter or by other means of transport are respectively $\frac{3}{10},\frac{1}{5},\frac{1}{10}$ *and* $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4},\frac{1}{3},\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means, he will not be late. He is late when he arrives. What is the probability that he comes by train?

OR

Find the probability distribution of the number of white balls drawn in a random draw of 3 balls without replacement from a bag containing 4 white and 6 red balls. Also find the mean and variance of the distribution.

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