BHARATIYA VIDYA BHAVAN'S V.M.PUBLIC SCHOOL, VADODARA

STD: XII **SAMPLE PAPER: 7** **MAX MARKS:100**
MATHEMATICS **TIME : 3 HRS**

General Instructions:

- All questions are compulsory
- The question paper consists of 29 questions divided into three sections A, B , C and D.
- Section A contains 4 questions of 1 mark each.
- Section B contains 8 questions of 2 marks each.
- Section C contains 11 questions of 4 marks each.
- Section D contains 6 questions of 6 marks each

SECTION A

1. Let f: R $\rightarrow$ R and g: R $\rightarrow$ R be two functions defined as f(x)= $|x|$+ x and g(x) = $|x| -$ x ,$\forall\, x \in$ R. find (g o f)(-2).
2. On the set Q-{1}, a binary operation * is defined by a * b = a + b – ab for all a, b $\in$ Q–{1}. Find the identity element and inverse of 'a' for each a $\in$ Q–{1}.
3. Find the values of a and b, if matrix A = $\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is skew-symmetric.
4. If $\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a} \,.\, \vec{b}\right|^2$= 400 and $|\vec{a}|$= 5, then write the value of $\left|\vec{b}\right|$.

SECTION B

5. Solve for x : cos $(\tan^{-1} x)$ = sin$\left[\cot^{-1}\left(\frac{3}{4}\right)\right]$
6. If adj A = $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and adj B = $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$, find adj (AB).
7. Verify Lagrange's Mean value theorem for the function f(x) = x + $\frac{1}{x}$ in the interval [1,3].
8. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, find the approximate error in calculating its surface area.
9. Evaluate $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$ using properties of integrals.
10. Form the differential equation of the family of circles touching the y-axis at origin.
11. The two adjacent sides of a parallelogram are 2$\hat{\imath}$- 4$\hat{\jmath}$ + 5$\hat{k}$ and $\hat{\imath}$ - 2$\hat{\jmath}$ - 3$\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find its area.
12. A random variable X has the following probability distribution: Determine P (x $\geq$ 6).

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2$+k

SECTION C

13. Find the inverse of A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$ using elementary transformations.

14. If y $\sqrt{x^2+1}$ = log $\left(\sqrt{x^2+1} - x\right)$, show that $(x^2+1)\frac{dy}{dx}$ + xy+1 = 0.

15. Discuss the continuity of the function $f(x) = \begin{cases} \frac{x^4+2x^3+x^2}{\tan^{-1} x} & x \neq 0 \\ 0, & x = 0 \end{cases}$ at x = 0.

OR

15. Find the values of a and b, if the function f defined by $f(x) = \begin{cases} x^2 + 3x + a, & x \le 1 \\ bx + 2, & x > 1 \end{cases}$ is differentiable at x = 1.

16. Find the area of the triangle formed by the tangent and the normal at the point (a,a) on the curve $y^2(2a-x) = x^3$ and the line x = 2a.

OR

16. Find the interval in which the function f(x) = $\frac{4x^2+1}{x}$ is (a) increasing (b) increasing.

17.Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone

is half of that of the cone.

18. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{sinx+cosx}{\sqrt{sin2x}}$ dx using properties of integrals.

19. Find the particular solution of the differential equation $\frac{dy}{dx}$+ ycotx = 4x cosecx ($x \neq 0$)given that y = 0 when x = $\frac{\pi}{2}$.

OR

19. Find the particular solution of the differential equation $2ye^{\frac{x}{y}}dx + (y-2xe^{\frac{x}{y}})dy = 0$ given that y = 1when x = 0.

20. The magnitude of the vector product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with the unit vector along the sum of vectors

$\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and$\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ.

21. Find the shortest distance between the lines whose vector equations are
$\vec{r} = (\hat{\imath} + \hat{\jmath}) + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} + \hat{\jmath} - \hat{k}) + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$.

22. In a self-assessment survey 60% persons claimed that they never indulged in corruption, 40% persons claimed that they always spoke the truth and 20% said that they neither indulged in corruption nor told lies. A person is selected at random out of this group. (i)What is the probability that the person is either corrupt or tells lie? (ii) If the person never indulged in corruption, find the probability that she/he speaks the truth.(iii) If the person always speaks the truth then find the probability that she/he claims to have never indulged in corruption (iv) What values have been discussed in this question and why is it must for all to practice these values in our life?

23. An experiment succeeds twice often as it fails.find the probability that in the next 6 trials there will be at least 4 successes.

SECTION D

24. Let N be the set of all natural numbers and R be the relation on NxN defined by(a,b)R(c,d) iff ad(b+c) = bc(a+d). Prove that R is an equivalence relation.

OR

24. Let f: N → N be a function defined as f(x) = 9x^2 + 6x – 5. Show that f: N → S, where S is the range of f, is invertible.
Find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$.

25. If none of a, b, c is zero, using properties of determinants, prove

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (bc+ca+ab)^3$$

26. Find the area of that part of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

OR

26. Using integration, find the area of the given region after making a rough sketch. $\{(x, y): |x-1| \le y \le \sqrt{5-x^2}\}$

27. Evaluate $\int \frac{(x+2)}{\sqrt{(x-2)(x-3)}}$ dx **OR** Evaluate: $\int \left\{\log(log x) + \frac{1}{(log x)^2}\right\}$dx

28. Find the vector equation of the plane passing through the points (2,1,-1) and (-1,3,4) and perpendicular to the plane x −2y + 4z = 10. Also show that the plane thus obtained contains the line $\vec{r} = (-\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) + \lambda(3\hat{\imath} - 2\hat{\jmath} - 5\hat{k})$.

29. In a mid day meal programme, an NGO wants to provide vitamin rich diet to the students of one school. The dietician wishes to mix together two types of food X and Y in such a way that the mixture contains atleast 8 units of vitamin A and 10 units of vitamin C. Food X contains 2 units/kg of vitamin A and 1units/kg of vitamin C. Food Y contains 1 unit/kg of vitamin A and 2 units/kg of C. One Kg of food X costs Rs. 50 andone Kg of food Y costs Rs. 70. Formulate the problem as LPP and solve it graphically for the minimum cost of such a mixture.

****

Prepared by Ms.S.Devasena