BHARATIYA VIDYA BHAVAN'S V.M.PUBLIC SCHOOL, VADODARA

STD: XII **SAMPLE PAPER: 6** **MAX MARKS:100**

MATHEMATICS **TIME : 3 HRS**

General Instructions:

- All questions are compulsory
- The question paper consists of 29 questions divided into three sections A, B , C and D.
- Section A contains 4 questions of 1 mark each.
- Section B contains 8 questions of 2 marks each.
- Section C contains 11 questions of 4 marks each.

SECTION - A

1. Show that three vectors $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} - 3\vec{j} - 5\vec{k}$ and $\vec{c} = 3\vec{i} - \overrightarrow{4j} - 4\vec{k}$ are coplanar.
2. If A is a square matrix of order 3 such that $|adj\ A|$ = 225, find $|A'|$.
3. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}|$ = 3, $|\vec{b}|$ = 4 and $|\vec{a} \times \vec{b}|$ = 6.
4. Write the general solution of the differential equation $x\frac{dy}{dx} = y$.

SECTION – B

5. Form the differential equation representing the given family of curves by eliminating arbitrary constants a and b: $\frac{x}{a} + \frac{y}{b} = 1$
6. If x = a (cost + t sint) and y = a (sint – t cost), t ∈ (0, $\frac{\pi}{2}$). Find $\frac{d^2 y}{dx^2}$.
7. Verify Lagrange's mean value theorem for f(x) = (x-1)(x-2)(x-3) in [1,4].
8. Find the projection of vectors $\vec{a}\ on\ \vec{b}\ if\ \vec{a} \cdot \vec{b} = 8\ \ and\ \vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$.
9. Using matrices, solve the system of equations : x + 2y + z = 7; x + 3z = 11 ; 2x – 3y =1.
10. Express the given matrix as the sum of a symmetric and skew symmetric matrix and verify your result.
$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$
11. Prove: $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$
12. If f:R→R and g:R→R be functions defined by f(x) = $[x]$ and g(x) = $|x|$, then evaluate gof $\left(\frac{7}{3}\right) + fog\left(\frac{7}{3}\right)$.

SECTION C

13. Using properties of determinants , prove $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$.
14. Given that $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & , x < 0 \\ a & , x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & , x > 0 \end{cases}$. if f(x) is continuous at x = 0 , find a .
15. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}, prove\ (1-x^2)\ y'' - 3x\ y' - y = 0$
16. Evaluate : $\int \frac{2+\sin x}{1+\cos x} e^{x/2}$ dx.
17. Evaluate: $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\ dx.$
18. Evaluate: $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx.$

19. Evaluate $\int_0^{\pi/4} \log(1+\tan x)\,dx$ using properties of integrals.

OR

Evaluate $\int_1^2 (x^2 + x + 2)\,dx$ as limit of sums.

20. If $\vec{a} = 5\vec{i} - \vec{j} - 3\vec{k}$ and $\vec{b} = \vec{i} + 3\vec{j} - 5\vec{k}$, then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

21. Find the vector and Cartesian equation of the plane passing through the points A(2,2,-1), B(3,4,2) and C(7,0,6).

OR

Find the shortest distance between the lines whose vector equations are
$\vec{r} = (\vec{i} + 2\vec{j} + 3\vec{k}) + \lambda(\vec{i} - 3\vec{j} + 2\vec{k})$ and $\vec{r} = (4\vec{i} + 5\vec{j} + 6\vec{k}) + \mu(2\vec{i} + 3\vec{j} + \vec{k})$

22. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up tail 75% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

23. (i) Check whether the relation R defined in the set {1,2,3,4,5,6} as R = {(a,b): b = a+1} is reflexive, symmetric or transitive.

(ii) Let * be a binary operation on the set of all non-zero real numbers, given by a*b = $\frac{ab}{5}$ for all a,b ∈ R−{0}. Find the value of x given that 2*(x*5) =10.

SECTION D

24. Using method of integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

25. Find the particular solution, satisfying the given condition, for the following differential equation: $\frac{dx}{dy}$ + x coty = 2y + $y^2 coty$ = 0, y≠0, given that x = 0 when y = $\frac{\pi}{2}$.

OR

Show that the given differential equation $2ye^{\frac{x}{y}}\,dx + (y - 2xe^{\frac{x}{y}})dy = 0$ is homogeneous. Find the general solution.

26. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each economy class ticket. The airlines reserves at least 20 seats for the first class. However at least 4 times as many passengers prefer to travel by economy class than the first class. Determine how many of each type ticket must be sold in order to maximize the profit for the airline? What is the maximum profit? Form an LPP and solve graphically.

27. A box contains 13 bulbs out of which 5 bulbs are defective .3 bulbs are drawn one by one from the box with replacement .Find the probability distribution of the number of defective bulbs drawn. Also find mean and variance.

28. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes x+2y+3z = 5 and 3x+3y+z = 0.

29. Prove that the radius of the base of right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone.

OR

Show that the semi-vertical angle of right circular cone of a given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$

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