BHARATIYA VIDYA BHAVAN'S V.M.PUBLIC SCHOOL, VADODARA

STD: XII **SAMPLE PAPER: 5** **MAX MARKS:100**
MATHEMATICS **TIME : 3 HRS**

General Instructions:

- All questions are compulsory
- The question paper consists of 29 questions divided into three sections A, B , C and D.
- Section A contains 4 questions of 1 mark each.
- Section B contains 8 questions of 2 marks each.
- Section C contains 11 questions of 4 marks each.
- Section D contains 6 questions of 6 marks each.

SECTION A

1. In the group (Z, *) of all integers where a * b = a + b + 1 for a, b Z, then what is the inverse of – 2?
2. Using determinants find the value of k for which the following system of equations has unique solution,
   2x – 5y = 26 ; 3x + ky = 5 .
3. Evaluate the value of $\hat{i} \bullet (\hat{j} \times \hat{k}) + \hat{j} \bullet (\hat{k} \times \hat{i}) + \hat{k} \bullet (\hat{i} \times \hat{j})$,
4. If f: R $\rightarrow$R defined as f(x)=$\frac{2x-7}{4}$ is invertible function ,write f^{-1} (x).

SECTION B

5. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$,where xy < 1, find the value of x+y+ xy .
6. Express $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices .Verify your answer.
7. Find $\frac{dy}{dx}$, if xy = $e^{(x-y)}$.
8. If x changes from 4 to 4.01, then find the approximate change in $\log_e x$.
9. Evaluate: $\int \frac{e^x (1+x)}{cos^2 (e^x x)}$ dx
10. Form the differential equation of the family of curves y = a sin (bx + c) , a, b, c are arbitrary constants.
11. If the sum of two unit vectors is a unit vector ,then show that the magnitude of their difference is $\sqrt{3}$.
12. If A and B are two independent events ,then show that the probability of occurrence of atleast one of A and B is given by 1 - P(A′) P(B′) .

SECTION C

13. If A = $\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ and f(x) = x^2 – 5x – 14,find f(A) .Hence obtain A^3 .
14. Find the value of k , if the function $f(x) = \begin{cases} \frac{2^{x+2}-16}{4^x-16}, & if\ x \neq 2 \\ k, & if\ x = 2 \end{cases}$ is continuous at x = 2.

OR

If 3 f (x) – 2 f (1/x) = x,, then find f ′ (2)

15. If x = secθ - cos θ and y = $\sec^n \theta - \cos^n \theta$,then show that $(x^2 + 4)(\frac{dy}{dx})^2 = n^2 (y^2 + 4)$.
16. Find the local maximum and local minimum, if any, for the function f(x) = $x^3 – 6x^2 + 9x + 15$. Also, find the local extreme values.

OR

Show that the line $\frac{x}{a} + \frac{y}{b} = 1$,touches the curve b $e^{-x/a}$ at the point where it crosses the y axis.

17. Water is dropping out at a steady rate of 1 cu. cm /sec through a tiny hole at the vertex of the conical vessel whose axis is vertical .When the slant height of water in the vessel is 4 cm ,find the rate of decrease of slant height where the semi vertical angle of the cone is $\frac{\pi}{6}$. Write any two measures to save water.

18. Evaluate $\int \frac{\cos x - \sin x}{5 + \sin 2x} dx$

19. If y(t) is a solution of (1 + t) $\frac{dy}{dx}$ – ty = 1 and y(0)= - 1 ,then show that y(1)= - ½

OR

Solve : x $\frac{dy}{dx}$ = y (log y – log x+1) .

20. Prove that $\vec{a} \bullet \left(\vec{b} \times \vec{c} \right) \times \left(\vec{a} + \vec{b} + \vec{c} \right) = 0$.

21. Find the value of λ so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

22. A car manufacturing factory has two plants X and Y. Plant X manufactures 70 % of the cars and plant Y manufactures 30 % . 80 % of the cars at plant X and 90 % of the cars at plant Y are rated of standard quality. A car is chosen at random and found to be of standard quality. What is the probability that it has come from plant X ?

23. Two cards are drawn simultaneously from a pack of 52 cards. Find the mean and variance of the number of red cards .

Section D

24. Consider the binary operations * and # on R defined as a* b = | a –b | and a# b = a for all a, b in R. Show that * is commutative but not associative ,# is associative but not commutative. Also show that a * (b # c) =(a * b) # (a * c) for all a ,b, c $\in$ R . Is a # (b * c) =(a # b)*(a # c) true for all a, b, c $\in$R ? Justify your answer.

OR

Find the inverse of the function f: N $\rightarrow$ N defined by f(x)= x^2 + x + 1, x $\in$ N, if it exists .

25. Using properties of determinants ,prove $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$.

OR

Using properties of determinants ,solve the following equation for x: $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$

26. Using integration find the area of the region bounded by the following curves after making a rough sketch: y= 1+| x +1 | , x = - 3 , x = 3 and y = 0.

27. Evaluate : $\int_0^1 \cot^{-1}\left(1 - x - x^2\right) dx$ OR Evaluate as the limit of sums $\int_1^4 \left(x^2 - x\right) dx$.

28. Find the length and foot of perpendicular from the point (1, 3/2 , 2) to the plane 2 x – 2 y +4 z + 5 =0 .

29. A factory owner purchases two types of machines A and B for his factory.The requirements and limitations for the machines are as follows:

machine	Area occupied by the machine	Labour force for each machine	Daily output (in units)
A	1000 sq. m	12 men	60
B	1200 sq. m	8 men	40

He has an area of 9000 sq. available and 72 skilled men who can operate the machines. How many machines of each type should he buy to maximise the daily output?