BHARATIYA VIDYA BHAVAN'S V.M.PUBLIC SCHOOL, VADODARA

STD: XII **SAMPLE PAPER: 1** **MAX MARKS:100**
SUBJECT : MATHEMATICS **TIME : 3 HRS**

General Instructions:

- All questions are compulsory
- The question paper consists of 29 questions divided into three sections A, B , C and D.
- Section A contains 4 questions of 1 mark each.
- Section B contains 8 questions of 2 marks each.
- Section C contains 11 questions of 4 marks each.
- Section D contains 6 questions of 6 marks each.

Section A

1. Write the principal value of $\tan^{-1}[\sin(-\pi/2)]$.
2. The Cartesian equations of a line are $6x-2=3y+1=2z-2$. Find its direction cosines.
3. Let R be the equivalence relation in the set A = { 0 , 1 , 2 , 3 , 4 , 5] give by R = { (a , b): 2 divides (a – b)}. Write the equivalence class [0].
4. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x^2 + y^2$.

Section B

5. If f(x) = x^2 + 1 and g(x) = $\frac{1}{x-1}$, find gof (-5).
6. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x.
7. Write the derivative of sin x w.r.t. cos x
8. Find the sum of order and degree of the following differential equation

$$5x\frac{dy}{dx}-\left(\frac{d^2y}{dx^2}\right)^3=-6y\,.$$

9. Evaluate: P (A U B); if 2 P(A) = P(B) = $\frac{5}{13}$ and P (A/B) = $\frac{2}{5}$.

10. Solve graphically :maximize z= 3x +4y subject to x + y ≤ 4, x,y ≥ 0.

11. .A man is known to speak the truth 3 out of 5. He throws a die and reports that it is 1, find the probability that it is actually 1.
12. The surface area of a spherical bubble is increasing at the rate of $2cm^2$/sec. Find the rate at which the volume of the bubble is increasing at the instant if its radius is 6 cm.
13. Using the properties of the determinants, prove that:

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

OR

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab + bc + ca + abc$$

14.. If $y = \tan^{-1}\frac{a}{x} + \log\sqrt{\frac{x-a}{x+a}}$, prove that $\frac{dy}{dx} = \frac{2a^3}{x^4 - a^4}$.

15. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is
(i) Strictly increasing (ii) Strictly decreasing

OR

Find the equations of the tangent and normal to the curve $x = a\sin^3\theta$ and $y = a\cos^3\theta$ at $\theta = \frac{\pi}{4}$.

16. Evaluate: $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx$

17.. Find the area of the region enclosed between the two circles:
$x^2 + y^2 = 1$ and $(x+1)^2 + y^2 = 1$

18. Find the particular solution of the differential equation:
$x\frac{dy}{dx} - y + x\,\text{cosec}\frac{y}{x} = 0$, given that $y = 0$ when $x = 0$.

OR

. $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x$, $x \neq 0$, given that $y = 0$. When $x = \frac{\pi}{2}$.

19.. If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = 2\hat{j} - \hat{k}$ are three vectors , find the area of the parallelogram having diagonals ($\vec{a} + \vec{b}$) and ($\vec{b} + \vec{c}$).

20. Find the equation of the line which is perpendicular from the point (3, - 1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also find the foot of the perpendicular and length of the perpendicular.

21. If a young man rides his motor cycle at 25 kmperhour, he had to spend Rs 2per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40km per hour and rate of pollution also increases, he has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as a L.P.P., and solve it graphically to find the distance to be covered with different speeds. What value is indicated in this question?

22. A class has 15 students whose ages are 14,17,15,14,21,17,19,20,16,18,20,17,16,19 and 20 years. One student is selected in such a manner that each has the same

chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find the mean of X.

23. Raj is taking up 3 subjects Math, Physics and Chemistry in the examination. His probability of getting grade A in these subjects is 0.2, 0.3 and 0.5 respectively. Find the probability that he gets grade A in (i) all subjects.(ii) no subjects.(iii) two subjects.

24. Evaluate $\int_a^b sinx$ dx as limit of sum.

OR

Evaluate: $\int \frac{1}{\sin^4 x + \cos^2 x \sin^2 x + \cos^4 x} dx$

25. Prove that $\cos^{-1} x + \cos^{-1} \left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right] = \frac{\pi}{3}$.

OR

Solve for x : $\tan^{-1} x + 2 \cot^{-1} x = \frac{2\pi}{3}$

26. Two factories decided to award their employees for three values of (a) adaptable to new techniques, (b) careful and alert in difficult situations and (c) keeping calm in tense situations at the rate of Rs x ,Rs y and Rs z per person respectively. The first factory decided to honour respectively 2 , 4 , 3 employees with a total prize money of Rs 29000. The second factory decided the honour respectively 5, 2 and three employees with the prize money of Rs 30,500. If the three prizes per person together cost Rs 9500 then, represent the above situation by matrix equations and solve these equations by matrix method

OR

Find A^{-1} using elementary transformations if $A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 1 \\ 1 & 6 & 3 \end{bmatrix}$.

27. Show that the semi vertical angle of the cone of time maximum volume and the given slant height is $\cos^{-1}\left(\frac{1}{3}\right)$

28. Let N be the set of all natural numbers and R be the relation on N x N defined by (a,b) R (c,d) if and only if, ad (b + c) = bc (a + d). Show that R is an equivalence relation.

29. Find the shortest distance between the two lines whose vector equations are

$$\vec{r} = \left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right) + \lambda\left(\hat{\imath} - 3\hat{\jmath} + 2\hat{k}\right) \text{ and } \vec{r} = \left(4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{\jmath} + \hat{k}\right)$$

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